

New Types of Continuous Maps and Hausdorff Spaces in Nano Ideal Spaces

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Abstract- In this paper, we introduce new concepts of continuous maps and lies between nano Hausdorff spaces and $n\mathcal{J}_g$ -Hausdorff spaces in nano ideal spaces.

Key Words and Phrases- $n\mathcal{J}_g$ - *-continuous map, $n\mathcal{J}_{g\#}$ - *-continuous map, nano Hausdorff spaces, nano *-Hausdorff spaces and $n\mathcal{J}_g$ -Hausdorff spaces.

1. INTRODUCTION

In 1970, Levine [7] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. M. K. R. S. Veera kumar [13] introduced a new class of sets, namely $g^\#$ -closed sets in topological spaces.

An ideal \mathcal{J} on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

- (1) $A \in \mathcal{J}$ and $B \subseteq A \Rightarrow B \in \mathcal{J}$ and
- (2) $A \in \mathcal{J}$ and $B \in \mathcal{J} \Rightarrow A \cup B \in \mathcal{J}$.

Given a topological space (X, τ) with an ideal \mathcal{J} on X and if $\wp(X)$ is the set of all subsets of X , a set operator $(.)^* : \wp(X) \rightarrow \wp(X)$, called a local function [4] of A with respect to τ and \mathcal{J} is defined as follows: for $A \subseteq X$, $A^*(\mathcal{J}, \tau) = \{x \in X : U \cap A \notin \mathcal{J} \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$. We will make use of the basic facts about the local functions ([3], Theorem 2.3) without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology

$\tau^*(\mathcal{J}, \tau)$, called the *-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\mathcal{J}, \tau)$ [12]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{J}, \tau)$ and τ^* for $\tau^*(\mathcal{J}, \tau)$. If \mathcal{J} is an ideal on X , then (X, τ, \mathcal{J}) is called an ideal topological space. A subset A of an ideal topological space (X, τ, \mathcal{J}) is *-closed [3] if $A^* \subseteq A$. The interior of a subset A in $(X, \tau^*(\mathcal{J}))$ is denoted by $int^*(A)$.

In this paper, we introduce new concepts of continuous maps and lies between nano Hausdorff spaces and $n\mathcal{J}_g$ -Hausdorff spaces in nano ideal spaces.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $n-cl(A)$ and $n-int(A)$ denote the nano closure of A and the nano

interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 [11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x .

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [5] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets (briefly n-open sets) and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 [5] If $[\tau_R(X)]$ is the nano topology on U with respect to X , then the set $B = \{U, \emptyset, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 [5] If $(U, \tau_R(X))$ is a nano topological space with respect to U and if $A \subseteq U$, then the nano interior of A is defined as the union of all n-open subsets of A and it is denoted by $n\text{-int}(A)$.

That is, $n\text{-int}(A)$ is the largest n-open subset of A . The nano closure of A is defined as the intersection of all n-closed sets containing A and it is denoted by $n\text{-cl}(A)$.

That is, $n\text{-cl}(A)$ is the smallest n-closed set containing A .

Definition 2.6 [2] An ideal \mathcal{I} on a nano topological space (X, τ) is a nonempty collection of subsets of X which satisfies

- (1) $A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$ and
- (2) $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.

Definition 2.7 [6] A nano topological space $(U, \tau_R(X))$ with an ideal \mathcal{I} on U is called a nano ideal topological space or nano ideal space and denoted as $(U, \tau_R(X), \mathcal{I})$.

Definition 2.8 [6] Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space. If $\wp(X)$ is the set of all subsets of U , a set operator $(.)^* : \wp(U) \rightarrow \wp(U)$ is called the

nano local function of \mathcal{J} on U with respect to \mathcal{J} on $\tau_R(X)$ is defined as $A_n^* = \{x \in U: U \cap A \notin \mathcal{J}, \text{ for every } U \in \tau_R(X)\}$ and is denoted by A_n^* , where nano closure operator is defined as $n\text{-cl}^*(A) = A \cup A_n^*$.

Result 2.9 [6] Let $(U, \tau_R(X), \mathcal{J})$ be a nano ideal topological space and A and B be subsets of U . Then

1. $A \subseteq B \Rightarrow A_n^* \subseteq B_n^*$,
2. $A_n^* = n\text{-cl}(A_n^*) \subseteq n\text{-cl}(A)$ (A_n^* is a n -closed subset of $n\text{-cl}(A)$),
3. $(A_n^*)_n^* \subseteq A_n^*$,
4. $(A \cup B)_n^* = A_n^* \cup B_n^*$,
5. For every nano open set $V \Rightarrow V \cap A_n^* = V \cap (V \cap A)_n^* \subseteq (V \cap A)_n^*$,
6. $J \in \mathcal{J} \Rightarrow (A \cup J)_n^* = A_n^* = (A - J)_n^*$

Result 2.10 [6] Let $(U, \tau_R(X), \mathcal{J})$ be a nano ideal topological space and A be a subset of U , If $A \subseteq A_n^*$, then $A_n^* = n\text{-cl}(A_n^*) = n\text{-cl}(A) = n\text{-cl}^*$.

Theorem 2.11 [10] In a space $(U, \tau_R(X), \mathcal{J})$, if A and B are subsets of U , then the following results are true for the set operator $n\text{-cl}^*$.

1. $A \subseteq n\text{-cl}^*(A)$,
2. $n\text{-cl}^*(\emptyset) = \emptyset$ and $n\text{-cl}^*(U) = U$,
3. If $A \subset B$, then $n\text{-cl}^*(A) \subseteq n\text{-cl}^*(B)$,
4. $n\text{-cl}^*(A) \cup n\text{-cl}^*(B) = n\text{-cl}^*(A \cup B)$,
5. $n\text{-cl}^*(n\text{-cl}^*(A)) = n\text{-cl}^*(A)$.

Definition 2.12 A subset A of a nano topological space $(U, \tau_R(X))$ is called nano α -open [5] if $A \subseteq n\text{-int}(n\text{-cl}(n\text{-int}(A)))$.

The complements of nano α -open set is called nano α -closed set.

Definition 2.13 [1] A subset A of a nano topological space $(U, \tau_R(X))$ is called nano α g-closed [8] if $n\text{-acl}(A) \subseteq G$ whenever $A \subseteq G$ and G is n -open.

The complements of nano α g-closed set is called

nano α g-open set.

Definition 2.14 A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{J})$ is called nano $*$ -open [9] if $A_n^* \subseteq A$.

The complements of nano $*$ -open set is called nano $*$ -closed set.

Definition 2.15 A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{J})$ is called nano \mathcal{J} -g-closed (briefly, $n\mathcal{J}_g$ -closed) [9] if $A_n^* \subseteq G$ whenever $A \subseteq G$ and G is n -open.

The complements of $n\mathcal{J}_g$ -closed set is called $n\mathcal{J}_g$ -open set.

3. NEW TYPES OF CONTINUOUS MAPS IN NANO IDEAL SPACES

Definition 3.1 A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{J})$ is called nano $\mathcal{J}_{g\#}$ -closed (briefly, $n\mathcal{J}_{g\#}$ -closed) if $A^* \subseteq G$ whenever $A \subseteq G$ and G is nano α g-open.

The complements of $n\mathcal{J}_{g\#}$ -closed set is called $n\mathcal{J}_{g\#}$ -open set.

Example 3.2 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ and $X = \{a, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, d\}, \{a, b, d\}\}$, $\mathcal{J} = \{\emptyset\}$. It is clear that $\{c\}$ is $n\mathcal{J}_{g\#}$ -closed set.

Definition 3.3 A map $f: (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y))$ is said to be

1. $n\mathcal{J}_g$ -continuous if $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U for every n -closed set F of V ;
2. $n\mathcal{J}_{g\#}$ -continuous if $f^{-1}(F)$ is $n\mathcal{J}_{g\#}$ -closed in U for every n -closed set F of V .

Example 3.4 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c\}\}$ and $\mathcal{J} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Let $V = \{a, b, c\}$ with $V/R = \{\{b\}, \{a, c\}\}$ and $Y = \{c\}$. Then $\tau_R(Y) = \{\emptyset, V, \{a, c\}\}$. Define $f: (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y))$ is defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Then f is $n\mathcal{J}_g$ -continuous map.

Example 3.5 In Example 3.4. Then f is

$n\mathcal{J}_g^{\#}$ -continuous map.

Definition 3.6 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is said to be

1. $n\mathcal{J}_g^{\#}$ -continuous if $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U for every nano $*$ -closed set F of V ;
2. $n\mathcal{J}_g^{\#}$ -continuous if $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U for every nano $*$ -closed set F of V .

Example 3.7 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c\}\}$ and $\mathcal{J} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Let $V = \{a, b, c\}$ with $V/R = \{\{b\}, \{a, c\}\}$ and $Y = \{c\}$. Then $\tau_R(Y) = \{\emptyset, V, \{a, c\}\}$ and $\mathcal{J} = \{\emptyset, \{c\}\}$. Define $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is defined by $f(a) = a, f(b) = b, f(c) = c$. Then f is $n\mathcal{J}_g^{\#}$ -continuous map.

Example 3.8 In Example 3.7. Then f is $n\mathcal{J}_g^{\#}$ -continuous map.

Definition 3.9 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is said to be

1. $n\mathcal{J}_g$ -irresolute if $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U for every $n\mathcal{J}_g$ -closed set F of V ;
2. $n\mathcal{J}_g^{\#}$ -irresolute if $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U for every $n\mathcal{J}_g^{\#}$ -closed set F of V .

Example 3.10 In Example 3.7. Then f is $n\mathcal{J}_g$ -irresolute map.

Example 3.11 In Example 3.7. Then f is $n\mathcal{J}_g^{\#}$ -irresolute map.

Theorem 3.12 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y))$ is $n\mathcal{J}_g$ -continuous if and only if $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U for every n -closed set F of V .

Proof. Suppose that f is $n\mathcal{J}_g$ -continuous map. Let F be any n -closed set of V . By assumption of f , $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U .

Conversely, suppose that the condition holds. Let F be any n -closed set of V . By the given condition, $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U . Hence f is $n\mathcal{J}_g$ -continuous map.

Theorem 3.13 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V,$

$\tau_R(Y))$ is $n\mathcal{J}_g^{\#}$ -continuous if and only if $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U for every n -closed set F of V .

Proof. Suppose that f is $n\mathcal{J}_g^{\#}$ -continuous map. Let F be any n -closed set of V . By assumption of f , $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U .

Conversely, suppose that the condition holds. Let F be any n -closed set of V . By the given condition, $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U . Hence f is $n\mathcal{J}_g^{\#}$ -continuous map.

Theorem 3.14 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is $n\mathcal{J}_g$ -irresolute if and only if $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U for every $n\mathcal{J}_g$ -closed set F of V .

Proof. Suppose that f is $n\mathcal{J}_g$ -irresolute map. Let F be any $n\mathcal{J}_g$ -closed set of V . By assumption of f , $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U .

Conversely, suppose that the condition holds. Let F be any $n\mathcal{J}_g$ -closed set of V . By the given condition, $f^{-1}(F)$ is $n\mathcal{J}_g$ -closed in U . Hence f is $n\mathcal{J}_g$ -irresolute map.

Theorem 3.15 A map $f : (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is $n\mathcal{J}_g^{\#}$ -irresolute if and only if $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U for every $n\mathcal{J}_g^{\#}$ -closed set F of V .

Proof. Suppose that f is $n\mathcal{J}_g^{\#}$ -irresolute map. Let F be any $n\mathcal{J}_g^{\#}$ -closed set of V . By assumption of f , $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U .

Conversely, suppose that the condition holds. Let F be any $n\mathcal{J}_g^{\#}$ -closed set of V . By the given condition, $f^{-1}(F)$ is $n\mathcal{J}_g^{\#}$ -closed in U . Hence f is $n\mathcal{J}_g^{\#}$ -irresolute map.

4. NEW TYPES OF HAUSDORFF SPACES IN NANO IDEAL SPACES

we define new concepts of nano Hausdorff spaces, nano $*$ -Hausdorff spaces and $n\mathcal{J}_g$ -Hausdorff spaces.

Definition 4.1 A nano topological space $(U, \tau_R(X))$ is called nano Hausdorff if for every two different points x and y of U , there exist disjoint n -open sets P and Q of U such that $x \in P$ and $y \in Q$.

Definition 4.2 A nano ideal topological space $(U, \tau_R(X), \mathcal{J})$ is called nano $*$ -Hausdorff if for every two different points x and y of U , there exist disjoint nano $*$ -open sets P and Q of U such that $x \in P$ and $y \in Q$.

It is obvious that every nano Hausdorff space is nano $*$ -Hausdorff space.

The following Example shows that the converse is not true.

Example 4.3 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c\}\}$ and $\mathcal{J} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then U is nano $*$ -Hausdorff space but not nano Hausdorff space.

Definition 4.4 A nano ideal topological space $(U, \tau_R(X), \mathcal{J})$ is called $n\mathcal{J}_g$ -Hausdorff if for every two different points x and y of U , there exist disjoint $n\mathcal{J}_g$ -open sets P and Q of U such that $x \in P$ and $y \in Q$.

It is obvious that every nano $*$ -Hausdorff space is $n\mathcal{J}_g$ -Hausdorff space.

The following Example shows that the converse is not true.

Example 4.5 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c\}\}$ and $\mathcal{J} = \{\emptyset, \{b\}\}$. Then U is $n\mathcal{J}_g$ -Hausdorff space but not nano $*$ -Hausdorff space.

Theorem 4.6 Let $(U, \tau_R(X), \mathcal{J})$ be a nano ideal topological space and be nano Hausdorff. If $f: (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y))$ is injective and $n\mathcal{J}_g$ -continuous, then $(U, \tau_R(X), \mathcal{J})$ is $n\mathcal{J}_g$ -Hausdorff.

Proof. Let x and y be any two different points of U . Then $f(x)$ and $f(y)$ are different points of Y because f is injective. Since Y is nano Hausdorff, there exist disjoint n -open sets P and Q in Y containing $f(x)$ and $f(y)$ respectively. Since f is $n\mathcal{J}_g$ -continuous and $P \cap Q = \emptyset$, we have $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint $n\mathcal{J}_g$ -open sets in U such that $x \in f^{-1}(P)$ and $y \in f^{-1}(Q)$. Hence U is an $n\mathcal{J}_g$ -Hausdorff.

Theorem 4.7 Let $(U, \tau_R(X), \mathcal{J})$ be a nano ideal topological space and $(V, \tau_R(Y), \mathcal{J})$ be nano $*$ -Hausdorff. If $f: (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is injective and $n\mathcal{J}_g$ - $*$ -continuous, then $(U, \tau_R(X), \mathcal{J})$ is

$n\mathcal{J}_g$ -Hausdorff.

Proof. Let x and y be any two different points of U . Then $f(x)$ and $f(y)$ are different points of V because f is injective. Since V is nano $*$ -Hausdorff, there exist disjoint nano $*$ -open sets P and Q in V containing $f(x)$ and $f(y)$ respectively. Since f is $n\mathcal{J}_g$ - $*$ -continuous and $P \cap Q = \emptyset$, we have $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint $n\mathcal{J}_g$ -open sets in U such that $x \in f^{-1}(P)$ and $y \in f^{-1}(Q)$. Hence U is an $n\mathcal{J}_g$ -Hausdorff.

Remark 4.8 We have the following implications for properties of spaces.

$\text{nano Hausdorff} \rightarrow \text{nano } *- \text{Hausdorff} \rightarrow n\mathcal{J}_g\text{-Hausdorff}$

None of the above implications is reversible.

Theorem 4.9 Let $(U, \tau_R(X), \mathcal{J})$ be an nano ideal topological space and $(V, \tau_R(Y), \mathcal{J})$ be $n\mathcal{J}_g$ -Hausdorff. If $f: (U, \tau_R(X), \mathcal{J}) \rightarrow (V, \tau_R(Y), \mathcal{J})$ is injective and $n\mathcal{J}_g$ -irresolute, then $(U, \tau_R(X), \mathcal{J})$ is $n\mathcal{J}_g$ -Hausdorff.

Proof. Let x and y be any two different points of U . Then $f(x)$ and $f(y)$ are different points of V because f is injective. Since V is $n\mathcal{J}_g$ -Hausdorff, there exist disjoint $n\mathcal{J}_g$ -open sets P and Q in V containing $f(x)$ and $f(y)$ respectively. Since f is $n\mathcal{J}_g$ -irresolute and $P \cap Q = \emptyset$, we have $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint $n\mathcal{J}_g$ -open sets in U such that $x \in f^{-1}(P)$ and $y \in f^{-1}(Q)$. Hence U is a $n\mathcal{J}_g$ -Hausdorff.

5. CONCLUSION

In this paper, we define several new concepts of continuous maps and Hausdorff spaces in nano ideal spaces which lies between nano Hausdorff spaces and $n\mathcal{J}_g$ -Hausdorff spaces are discussed. This shall be extended in the future research with some applications.

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